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Spinning strings, cosmic dislocations, and chronology protection

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A massless scalar field is quantized in the background of a spinning string with cosmic dislocation. By increasing the spin density toward the dislocation parameter, a region containing closed timelike curves (CTCs) eventually forms around the defect. Correspondingly, the propagator tends to the ordinary cosmic string propagator, leading therefore to a mean-square field fluctuation, which remains well behaved throughout the process, unlike the vacuum expectation value of the energy-momentum tensor, which diverges due to a subtle mechanism. These results suggest that back reaction leads to the formation of a "horizon" that protects from the appearance of CTCs.

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Investigations on quantum theory around spinning defects go back to the late 1980s with the study of quantum mechanics of relativistic particles on the spinning cone [1]. Such a background is the Kerr-like solution of the Einstein equations in three dimensions, whose line element is given by (throughout the text $c = \hbar = 1$, and the metric parameters are nonnegative)

$$ds^2 = (d\tau + Sd\theta)^2 - dr^2 - \alpha^2 r^2 d\theta^2, \tag{1}$$

where S and α are the spin and the disclination parameter, respectively [2]. Clearly Minkowski spacetime corresponds to $S = 0$ and $\alpha = 1$. Lifting the geometry in Eq. (1) to four dimensions, one obtains the gravitational background around a spinning cosmic string [3], for which

$$ds^2 = (d\tau + Sd\theta)^2 - dr^2 - \alpha^2 r^2 d\theta^2 - d\xi^2. \tag{2}$$

An inspection of Eqs. (1) and (2) shows that the region for which $r < S/\alpha$ contains CTCs, resulting that when $S \neq 0$ the corresponding spacetimes are not globally hyperbolic. It is not clear if quantum theory makes sense in nonglobaly hyperbolic spacetimes [4]. In fact, quantum mechanics on the spinning cone has shown that $S \neq 0$ spoils unitarity [1]. In the context of the second quantization around spinning cosmic strings [5], a recent analysis has revealed that a nonvanishing spin density S leads to divergent vacuum fluctuations [6].

In order to recover boost invariance along the symmetry axis, the authors in Ref. [7] have "amended" the geometry in Eq. (2) by postulating a cosmic dislocation, such that

$$ds^2 = (d\tau + Sd\theta)^2 - dr^2 - \alpha^2 r^2 d\theta^2 - (d\xi + \kappa d\theta)^2, \tag{3}$$

whose metric tensor fits as solution of the Einstein equations, as well as solution of the Einstein-Cartan equations [8, 9]. When $S > \kappa$, the region for which $r < \sqrt{S^2 - \kappa^2}/\alpha$ contains CTCs. When $S < \kappa$ though, the spacetime is globally hyperbolic.

Vacuum fluctuations typically diverge on the Cauchy surface (chronology horizon), which separates a region with CTCs from another without CTCs (for a review see Ref. [10]). This fact has led to the chronology protection conjecture, according to which, physical laws do not allow the appearance of CTCs ("time machines") [11]. Although the geometry in Eq. (3) does not contain any Cauchy horizon [for $S > \kappa$, Eq. (3) describes an "eternal time machine"], it might be clarifying to study quantum effects in the corresponding spacetime as the metric parameters are adjusted such that CTCs are about to form. Using a massless scalar field as a probe, this work implements such an investigation by considering $S < \kappa$ and by taking $S \to \kappa$, i.e., arbitrarily close to the point when the spacetime is about to become nonglobally hyperbolic.

According to Ref. [7], when the metric parameters in Eq. (3) satisfy $S < \kappa$, a Lorentz frame exists with respect to which the spin density vanishes (and one might say, in this case, that $S \neq 0$ is a kinematic effect). Indeed, by performing the following Lorentz transformation in the $\tau - \xi$ plane,

$$t = \frac{\tau - v\xi}{\sqrt{1 - v^2}} \qquad z = \frac{\xi - v\tau}{\sqrt{1 - v^2}} \qquad v := S/\kappa, \tag{4}$$

Eq. (3) can be recast as

$$ds^2 = dt^2 - dr^2 - \alpha^2 r^2 d\theta^2 - (dz + \kappa' d\theta)^2, \tag{5}$$

describing the gravitational background of a cosmic dislocation with dislocation parameter

$$\kappa' := \sqrt{\kappa^2 - S^2}, \tag{6}$$

and for which the usual identification

$$(t, r, \theta, z) \sim (t, r, \theta + 2\pi, z) \tag{7}$$

is observed. It should be noted that the background of a spinning string can only be seen as that of a cosmic dislocation when $S < \kappa$, since when $S \geq \kappa$ the boost in Eq. (4) becomes singular.

Vacuum fluctuations of a massless scalar field ϕ around a cosmic dislocation have recently been reported in the literature [12]. Equations (5) and (7) show that when $\kappa' \to 0$ the corresponding vacuum fluctuations become those associated with an ordinary cosmic string (see, e.g., [13]). It follows that all scalar vacuum averages observed from the frame corresponding to Eq. (3) are meant to remain finite as S is taken arbitrarily close to the critical value κ. For example, as $S \to \kappa$ the mean-square field fluctuation approaches

$$\langle \phi^2(r) \rangle = \frac{1}{48\pi^2 r^2} \left(\alpha^{-2} - 1\right), \tag{8}$$

which is finite (away from the defect).

Turning to the vacuum expectation value of the energy-momentum tensor $\langle \mathcal{T}^{\mu}{}_{\nu} \rangle$, it is more convenient to use local inertial coordinates $(\mathrm{T}, r, \varphi, \Xi)$ and (t, r, φ, Z) associated with Eqs. (3) and (5), respectively, which are defined as $\mathrm{T} := \tau + S\theta$, $\varphi := \alpha\theta$, $\Xi := \xi + \kappa\theta$ and $Z := z + \kappa'\theta$. In terms of these coordinates, both Eqs. (3) and (5) become the Minkowski line element written in cylindrical coordinates, and Eq. (7) leads to $(t, r, \varphi, Z) \sim (t, r, \varphi + 2\pi\alpha, Z + 2\pi\kappa')$, revealing a "space-helical" structure. [Representing the spinning string by a rotating helix, Eq. (4) leads to the frame travelling through the symmetry axis, and for which the helix does not rotate]. It should be mentioned that when the coordinates $(\mathrm{T}, r, \varphi, \Xi)$ are used, the background appears to have also a "time-helical" structure [2].

Using Eq. (4), one finds out that the energy density $\left\langle \mathcal{T}^{\mathrm{T}}{}_{\mathrm{T}} \right\rangle$ in the spinning string inertial frame $(\mathrm{T}, r, \varphi, \Xi)$ is related with $\langle T^{\mu}{}_{\nu} \rangle$ in the cosmic dislocation inertial frame (t, r, φ, Z) by

$$\left\langle \mathcal{T}^{\mathrm{T}}{}_{\mathrm{T}} \right\rangle = \frac{\langle T^{t}{}_{t} \rangle - v^2 \left\langle T^{Z}{}_{Z} \right\rangle}{1 - v^2}. \tag{9}$$

A superficial investigation may suggest that the relativistic factor in Eq. (9) will make $\left\langle \mathcal{T}^{\mathrm{T}}{}_{\mathrm{T}} \right\rangle$ to diverge as $S \to \kappa$ [$v \to 1$, cf. Eq. (4)]. However, as will be seen shortly, this is incorrect — $\left\langle \mathcal{T}^{\mathrm{T}}{}_{\mathrm{T}} \right\rangle$ indeed diverges as $S \to \kappa$; but the mechanism through which that operates is rather subtle and does not involve any relativistic factor. Another pitfall consists of carrying over Eq. (9) the fact that $S \to \kappa$ [$\kappa' \to 0$, cf. Eq. (6)] leads to $\left\langle T^{Z}{}_{Z} \right\rangle \to \langle T^{t}{}_{t} \rangle$ [12], and then to conclude (incorrectly) that $\left\langle \mathcal{T}^{\mathrm{T}}{}_{\mathrm{T}} \right\rangle \to \langle T^{t}{}_{t} \rangle$. The flaw in this argument will be clear in the following.

At this point one recalls that $\langle T^{t}{}_{t} \rangle$ and $\left\langle T^{Z}{}_{Z} \right\rangle$ in Eq. (9) are vacuum fluctuations in the background of a cosmic dislocation with dislocation parameter κ'. As is explained in Ref. [12], $\langle T^{t}{}_{t} \rangle$ and $\left\langle T^{Z}{}_{Z} \right\rangle$ can be obtained by letting a certain differential operator to act on the corresponding renormalized propagator $D^{(\alpha,\kappa')}(x, \bar{x})$, according to the prescription in Eq. (17) of Ref. [12]. Proceeding along these lines, it follows that

$$\left\langle T^{Z}{}_{Z} \right\rangle - \left\langle T^{t}{}_{t} \right\rangle = -i \lim_{\bar{x} \to x} \left(\partial_t \partial_{\bar{t}} + \partial_Z \partial_{\bar{Z}}\right) D^{(\alpha,\kappa')}(x, \bar{x}), \tag{10}$$

and by letting the derivatives to act on the expression of $D^{(\alpha,\kappa')}(x, \bar{x})$ in Eq. (12) of Ref. [12], Eq. (10) yields

$$\left\langle T^{Z}{}_{Z} \right\rangle = \left\langle T^{t}{}_{t} \right\rangle + \frac{\kappa'^2}{r^6} f_\alpha\left(\kappa'^2/r^2\right), \tag{11}$$

where

$$f_\alpha(x) := -\frac{1}{2} \int_0^\infty d\lambda \sum_{n=1}^{\infty} \frac{n^2\left[\lambda^2 - \pi^2(4\alpha^2 n^2 - 1)\right]}{\left[\pi^2(2\alpha n + 1)^2 + \lambda^2\right]\left[\pi^2(2\alpha n - 1)^2 + \lambda^2\right]\left[\cosh^2(\lambda/2) + n^2\pi^2 x\right]^3}. \tag{12}$$

A quick power counting in Eq. (12) gives that $f_\alpha(x)$ diverges at $x = 0$ (when α is finite). A more careful analysis shows that $f_\alpha(x)$ diverges as $x \to 0$; but it does so mildly since $x f_\alpha(x) \to 0$.

By inserting Eq. (11) in Eq. (9) and recalling that $\kappa'^2 = \kappa^2(1-v^2)$, one ends up with

$$\left\langle \mathcal{T}^{\mathrm{T}}{}_{\mathrm{T}} \right\rangle = \left\langle T^{t}{}_{t} \right\rangle - \frac{S^2}{r^6} f_\alpha \left(\kappa'^2/r^2 \right). \tag{13}$$

Considering $S \to \kappa$ in Eq. (13), $\langle T^t{}_t \rangle$ approaches the ordinary cosmic string expression (and therefore remains finite), whereas the term carrying f_α diverges [one sees that the flaw mentioned above consists in manipulating improperly the numerator in Eq. (9) when κ' is very small]. One might say that $S = \kappa$ plays the role of a chronology horizon where mechanisms of chronology protection are expected to take place. The divergence in $\left\langle \mathcal{T}^{\mathrm{T}}{}_{\mathrm{T}} \right\rangle$ confirms this expectation. For completeness, the other components of $\langle \mathcal{T}^\mu{}_\nu \rangle$ are displayed below

$$\langle \mathcal{T}^\mu{}_\nu \rangle = \begin{pmatrix} \langle T^t{}_t \rangle - (S^2/r^6) f_\alpha & 0 & (S/\kappa') \left\langle T^Z{}_\varphi \right\rangle & (\kappa S/r^6) f_\alpha \\ 0 & \langle T^r{}_r \rangle & 0 & 0 \\ -(S/\kappa') \langle T^\varphi{}_Z \rangle & 0 & \langle T^\varphi{}_\varphi \rangle & (\kappa/\kappa') \langle T^\varphi{}_Z \rangle \\ -(\kappa S/r^6) f_\alpha & 0 & (\kappa/\kappa') \left\langle T^Z{}_\varphi \right\rangle & \left\langle T^Z{}_Z \right\rangle + (S^2/r^6) f_\alpha \end{pmatrix}, \tag{14}$$

with f_α evaluated at κ'^2/r^2. The approximate behavior of $\langle \mathcal{T}^\mu{}_\nu \rangle$ as $S \to \kappa$, at a given distance r from the defect, can be obtained from Eq. (14) by considering the expressions for $\langle T^\mu{}_\nu \rangle$ in Ref. [12]. For example, if ϕ is conformally coupled, it follows that

$$\langle \mathcal{T}^\mu{}_\nu \rangle = \frac{1}{r^4} \begin{pmatrix} -(S^2/r^2) f_\alpha - A & 0 & SB & (\kappa S/r^2) f_\alpha \\ 0 & -A & 0 & 0 \\ -SB/r^2 & 0 & 3A & \kappa B/r^2 \\ -(\kappa S/r^2) f_\alpha & 0 & \kappa B & (S^2/r^2) f_\alpha - A \end{pmatrix}, \tag{15}$$

where $A(\alpha) := (\alpha^{-4} - 1)/1440\pi^2$ and $B(\alpha)$ is defined as in Eq. (20) of Ref. [12] [$B(\alpha = 1) = 1/60\pi^2$]. Clearly the components that diverge as $S \to \kappa$ are those containing f_α. At this point it is pertinent to note that, as mentioned previously, $\left\langle \phi^2 \right\rangle$ remains well behaved as S approaches the critical value κ [see Eq. (8)]. If $f_\alpha(x)$ were not divergent at $x = 0$, $\langle \mathcal{T}^\mu{}_\nu \rangle$ would also remain finite as $S \to \kappa$ (and that would suggest violation of chronology protection).

The following are simple facts that help to figure out the physical implication of the results reported above. One begins by taking S infinitesimally smaller than κ, say $S = \kappa - \delta$. Then, a finite interval of proper time $\Delta\tau$ as measured in the spinning string frame [cf., Eq. (3)] would appear an arbitrarily large interval of time measured in the corresponding cosmic dislocation frame [cf., Eq. (5)], since the latter would be traveling nearly at the speed of light (recall that $v = S/\kappa$). If $\Delta\tau$ is the interval of proper time immediately before the emergence of a region containing CTCs (which corresponds to $\delta = 0$), it follows that such an event never would be detected in the cosmic dislocation frame. Moreover, as $\delta \to 0$, the semiclassical metric tensor in the cosmic dislocation frame approaches that found in Ref. [14], which presents mild contributions from back reaction, as is typically the case. Considering the divergences in the vacuum expectation value of the energy-momentum tensor, such mild contributions (and that is a crucial point) would be hugely amplified in the spinning string frame, suggesting the formation of a "horizon" when $\delta = 0$. Such a "horizon" would eventually prevent the appearance of CTCs around the spinning string.

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[1] P. S. Gerbert and R. Jackiw, Commun. Math. Phys. **124**, 229 (1989).
[2] S. Deser, R. Jackiw, and G. 't Hooft, Ann. Phys. **152**, 220 (1984).
[3] P. O. Mazur, Phys. Rev. Lett. **57**, 929 (1986).
[4] S. A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time* (Cambridge University Press, Cambridge, England, 1989).
[5] G. E. A. Matsas, Phys. Rev. D **42**, 2927 (1990).
[6] V. A. De Lorenci and E. S. Moreira, Jr., Phys. Rev. D **63**, 027501 (2001).
[7] D. V. Gal'tsov and P. S. Letelier, Phys. Rev. D **47**, 4273 (1993).
[8] K. P. Tod, Class. Quantum Grav. **11**, 1331 (1994).
[9] P. S. Letelier, Class. Quantum Grav. **12**, 471 (1995).
[10] M. Visser, gr-qc/0204022 (unpublished).

[11] S. W. Hawking, Phys. Rev. D **46**, 603 (1992).
[12] V. A. De Lorenci and E. S. Moreira, Jr., Phys. Rev. D **67**, 124002 (2003).
[13] A. G. Smith, in *The Formation and Evolution of Cosmic Strings*, Proceedings of the Cambridge Workshop, Cambridge, England, edited by G. W. Gibbons, S. W. Hawking, and T. Vachaspati (Cambridge University Press, Cambridge, England, 1990).
[14] W. A. Hiscock, Phys. Lett. B **188**, 317 (1987).